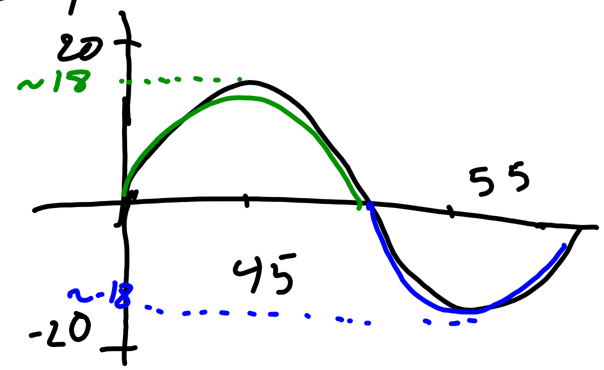
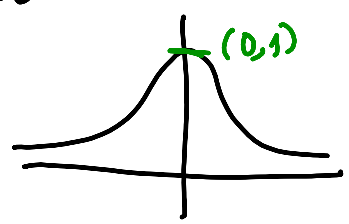
A51 #4

Given



Use $f(x) = \frac{1}{1+x^2}$ to describe the graph above.

$g(x) = \frac{1}{1+x^2}$



Top part of $f(x)$

$$f(x) = \frac{18}{1+(x-45)^2}$$

Bottom

$$f(x) = \frac{-18}{1+(x-55)^2}$$

Translations: To move up or down by a factor of c

$f(x)+c$, $f(x)-c$

To move left or right by c

$f(x+c)$, $f(x-c)$

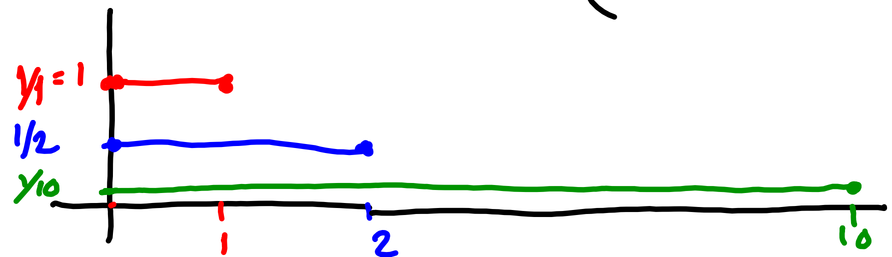
$c\,f(x)$ To scale the function by c

Consider $\mathrm{rect}_\ell = \begin{cases} 1/\ell & 0 \le x \le \ell \\ 0 & \text{otherwise} \end{cases}$

$\ell = 1 \quad 0 \le x \le 1$

$= 2 \quad 0 \le x \le 2$

$= 10$



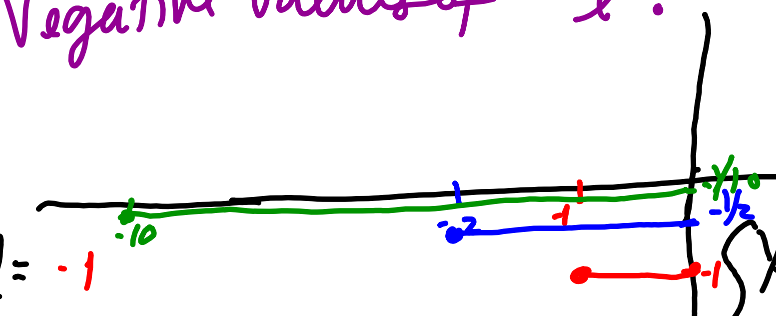
As ℓ goes to $+\infty$ the function is starting to look like $y=0$

Negative values of ℓ:

$\ell = -1$

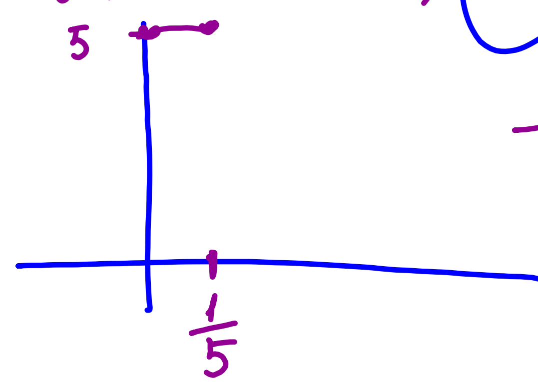
-2

-10



$\begin{cases} 1/\ell & -\ell \le x \le 0 \\ 0 & \text{else} \end{cases}$

look at $\ell = 5, 1/5, -\frac{1}{2}, \frac{1}{10}$



$$\frac{1}{\frac{1}{5}} = 5$$

Given the equation for turbidity $T = \frac{k S \ln(N)}{d^2}$

where N phytoplankton

S sediment

d is depth

a k some positive constant

T changes by what factor if depth and sediment is doubled?

$$T = \frac{k(2S)\ln(N)}{(2d)^2}$$

$$= \frac{2\,kS\ln(N)}{\cancel{2}^2\,d^2} = \frac{2}{4}(\ldots)$$

T changes by a factor of $\frac{1}{2}$

proportional to S

proportional to $\ln(N)$

$$T = \frac{k\,S\,\ln(N)}{d^2}$$

inverse squared proportion

On your own:

Write a function that's proportional to A, proportional to the inversion of $\ln(B)$, and proportional to C squared

D/R Review	Domain	Range
$\sin(x)$	All real	$[-1,1]$
$\cos(x)$	All real	$[-1,1]$
$\ln(x)$	$(0,\infty)$	All real numbers
e^x	All reals	$(0,\infty)$
$\sqrt{x}$	$[0,\infty)$	All reals
$\frac{1}{x}$	Everything but 0	Everything but 0

inverses

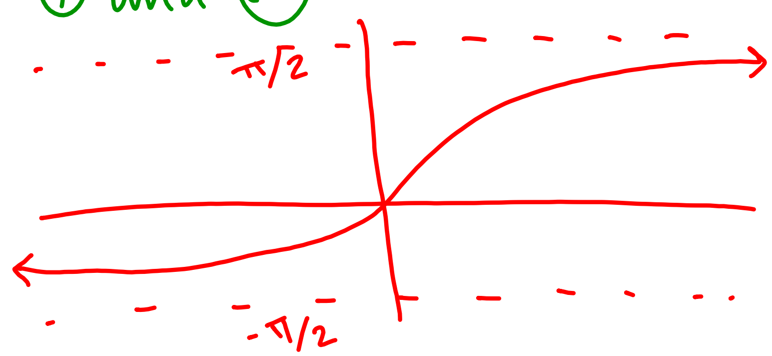
* Graphing has a restriction

* $\sqrt{4} = \pm 2$

1. $\arcsin(x) = (\sin^{-1}(x))$ $\quad [-\pi/2, \pi/2]$
2. $\arccos(x)$ $\quad [0, \pi]$
3. $\arctan(x)$ $\quad [-\pi/2, \pi/2]$

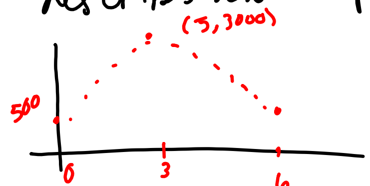
Domain picked $\sin x$, $\cos(x)$, $\tan x$

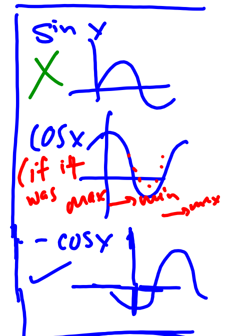
1 and 2 have restricted ranges



The population of bacteria in a lake changes periodically with a period of 6 days and

And is measured at the start of each week. Initially it has a min value of 500g and in 3 days it acheives a max of 3000g. By selecting the appropriate trig function find a formula which describes how the pop grows.





$V_0 + a\cos(\frac{1}{P}2\pi$

average

amplitude

freq'y

① Recognize which trig function works

②

Period: $\boxed{\frac{2\pi}{b}} = 6$

$2\pi = 6b \rightarrow \frac{2}{6}\pi = b, \ \frac{\pi}{3} = b$

Average: $\frac{\text{Max} + \text{Min}}{2}$

$1750 = \frac{3000 + 500}{2}$

Amplitude: $\frac{\text{Max} - \text{Min}}{2}$

$1250 = \frac{3000 - 500}{2}$

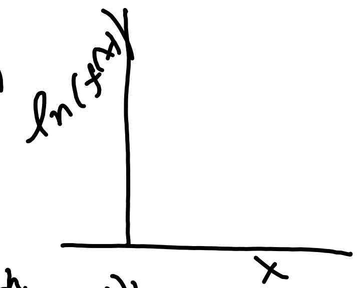
$1750 - 1250\cos(\frac{\pi}{3}t)$

On your own
Do the same problem except it acheives a max of 10,000 initially and a min of 200 (same time, same period)

Semi Log / Double Log

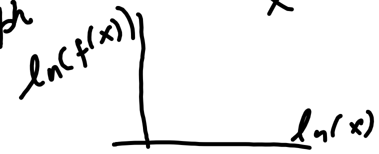
Semi Log graph

$f(x) = y$



Double Log graph

$f(x) = y$



Graph the double log function of kinetic energy $m = \frac{2E}{V^2}$ where

m = mass, V is velocity,

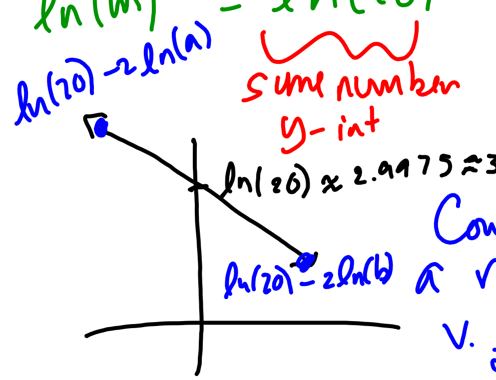
E is energy when E=10.

Consider $m = \frac{2(10)}{V^2} = \frac{20}{V^2}$

$$\ln(m) = \ln\left(\frac{20}{V^2}\right)$$

$$\ln(m) = \overset{\approx 3}{\ln(20)} - \ln(V^2)$$

$$\ln(m) = \underbrace{\ln(20)}_{\text{same number, y-int}} - \underbrace{2\ln(v)}_{\text{like a slope}}$$



Could specify a range for V. ie [a,b]

Given a function of population growth

$$M(t) = 8(1+e^{5t})$$

① Find $M^{-1}(t)$

② Given $H(t) = 3\ln(6t-7)$

Find $M(H(t))$

① $M(t) = 8(1+e^{5t})$

$$y = 8(1+e^{5x})$$

Often: switch y and x and solve for y

$$x = 8(1+e^{5y})$$

$$\frac{x}{8} = 1+e^{5y} \rightarrow \frac{x}{8}-1 = e^{5y}$$

$$\ln\left(\frac{x}{8}-1\right) = \ln(e^{5y})$$

$$= \ln\left(\frac{x}{8}-1\right) = 5y$$

$$\Rightarrow \frac{\ln\left(\frac{x}{8}-1\right)}{5} = y = M^{-1}(x)$$

②

$$H = 3\ln(6t-7)$$

$$M = 8(1+e^{5t})$$

Find $M(H(t))$

$$= 8(1+e^{5(H(t))})$$

$$= 8(1+e^{5(3\ln(6t-7))})$$

$$= 8(1+e^{15\ln(6t-7)})$$

$$= 8(1+\cancel{e^{\ln}}((6t-7)^{15}))$$

$$= 8(1+(6t-7)^{15})$$

e^x / ln Rules:

$$e^a \cdot e^b = e^{a+b}$$

$$\ln(a)+\ln(b) = \ln(ab)$$

$$e^a \div e^b = \frac{e^a}{e^b} = e^a \cdot e^{-b} = e^{a-b}$$

$$\ln(a)-\ln(b) = \ln(a)+\ln(b^{-1})$$
$$= \ln(a \cdot b^{-1}) = \ln\left(\frac{a}{b}\right)$$

$$e^{-x} = \frac{1}{e^x} \; ; \; \frac{1}{e^{-x}} = e^x$$

$$-\ln(x) = \ln(x^{-1}) = \ln\left(\frac{1}{x}\right)$$

$\ln(0)$ in undefined

$\ln(y)$ $y<0$ also undefined

$\ln(1) = 0$

$e^0 = 1$

$\log_e(1) = 0$

$e^{\boxed{0}} = 1$

$\log_a c = b$

$a^b = c$

$\log_e = \ln$